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Multidimensional quasi-linear diffusion of radiation belt electrons

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[1] We consider diffusion of outer zone radiation belt electrons by chorus waves. Quasi-linear diffusion coefficients valid outside the plasmasphere have only been calculated recently, and indicate that the energy and cross diffusion rates can be comparable to that for pitch angle diffusion. Proper solution of the diffusion equation for phase space density must therefore be based on the full diffusion tensor, but this has been plagued by numerical problems associated with the large and rapidly varying cross terms. To circumvent this, techniques are developed for transforming to variables in which the cross diffusion term vanishes. A model calculation shows significant diffusion of phase space density at $L = 4.5$ from 0.2 MeV up to a few MeV in less than a day. Citation: Albert, J. M., and S. L. Young (2005), Multidimensional quasi-linear diffusion of radiation belt electrons, *Geophys. Res. Lett.*, 32, L14110, doi:10.1029/2005GL023191.

1. Introduction

[2] A standard approach to studying the dynamics of energetic particles in Earth's radiation belts is via a quasi-linear diffusion equation, which describes the phase space distribution function suitably averaged over gyro-, bounce, and (longitudinal) drift frequencies. Electromagnetic waves resonant with these frequencies lead to diffusion in the three adiabatic invariants associated with them; other terms due to, e.g., Coulomb collisions are easily incorporated as well. In principle coupling occurs between the diffusion in all three invariants, but in a nearly axisymmetric dipole the diffusion in the third invariant J_3 (associated with waves at the drift frequency) is largely decoupled from diffusion in J_1 and J_2 (driven by waves resonant near or below the much higher cyclotron frequency). However, the diffusion in J_1 and J_2 is highly coupled, according to the formulation of Lyons [1974a, 1974b]. After bounce averaging, this treatment gives diffusion coefficients for equatorial pitch angle α_0 , momentum (or energy), and cross diffusion $D_{\alpha_0 p}$, which can be recast in terms of the adiabatic invariants [Schulz and Lanzerotti, 1974]. Then the diffusion equation takes the form

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial J_1} \left(D_{11} \frac{\partial f}{\partial J_1} + D_{12} \frac{\partial f}{\partial J_2} \right) + \frac{\partial}{\partial J_2} \left(D_{12} \frac{\partial f}{\partial J_1} + D_{22} \frac{\partial f}{\partial J_2} \right) + \frac{\partial}{\partial J_3} D_{33} \frac{\partial f}{\partial J_3}. \quad (1)$$

Because of the decoupling, the J_3 (or L or "radial") diffusion is relatively easy to treat, even if it is large.

[3] Inside the plasmasphere, waves cyclotron resonant with radiation belt electrons occur primarily as whistler mode hiss. There the plasma frequency f_{pe} is large compared to the cyclotron frequency f_{ce} , resulting in a pitch angle diffusion rate much greater than the energy diffusion rate, with the cross diffusion rate intermediate between the two [Lyons, 1974b]. Thus pure pitch angle diffusion combined with radial diffusion is a reasonable approximation there; further simplifying the pitch angle diffusion to a loss rate yields a successful description of the inner electron radiation belts, at least during quiet or moderately disturbed conditions [Lyons and Thorne, 1973]. Outside the plasmasphere, however, such models have not been able to reproduce the localized high energy phase space density peaks frequently observed to develop during the recovery phase of magnetic storms [e.g., Brautigam and Albert, 2000; Shprits and Thorne, 2004]. It has been argued that the condition $f_{pe} \sim f_{ce}$ enables storm-time enhanced whistler mode chorus to drive rapid electron acceleration [Summers et al., 1998; Horne et al., 2003, 2005; Albert, 2005], which could account for the energized population. Although the development of chorus waves involves strongly nonlinear processes [Sazhin and Hayakawa, 1992], we assume that quasi-linear theory is a meaningful description of the effect of fully developed chorus on the high energy population. Suggestive results have been obtained from a one dimensional energy diffusion equation averaged appropriately over α_0 [Summers and Ma, 2000; Summers et al., 2004; Horne et al., 2005; Li et al., 2005] but studying this process in detail requires treating the full velocity space diffusion equation. However, straightforward solution of equation (1) is elusive, even when the usual conditions for local, linear numerical stability are well satisfied, because of the large and rapidly varying cross diffusion coefficient. (See Albert [2004] for a review of previous approaches.) To overcome this, new variables are constructed for which the cross diffusion term vanishes, and standard finite difference techniques are applied to the transformed diffusion equation.

2. Eliminating the Cross Term

[4] In general, when transforming from (J_1, J_2) to new variables (Q_1, Q_2) , the diffusion matrix

$$D_{JJ} = \begin{bmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{bmatrix} \quad (2)$$

becomes [Haerendel, 1968; Schulz, 1991]

$$\tilde{D}_{QQ} = \begin{bmatrix} \partial Q_1 / \partial J_1 & \partial Q_1 / \partial J_2 \\ \partial Q_2 / \partial J_1 & \partial Q_2 / \partial J_2 \end{bmatrix} D_{JJ} \begin{bmatrix} \partial Q_1 / \partial J_1 & \partial Q_2 / \partial J_1 \\ \partial Q_1 / \partial J_2 & \partial Q_2 / \partial J_2 \end{bmatrix} \quad (3)$$

It is desired that the cross diffusion terms in the new variables vanish. It will be assumed that the diffusion is

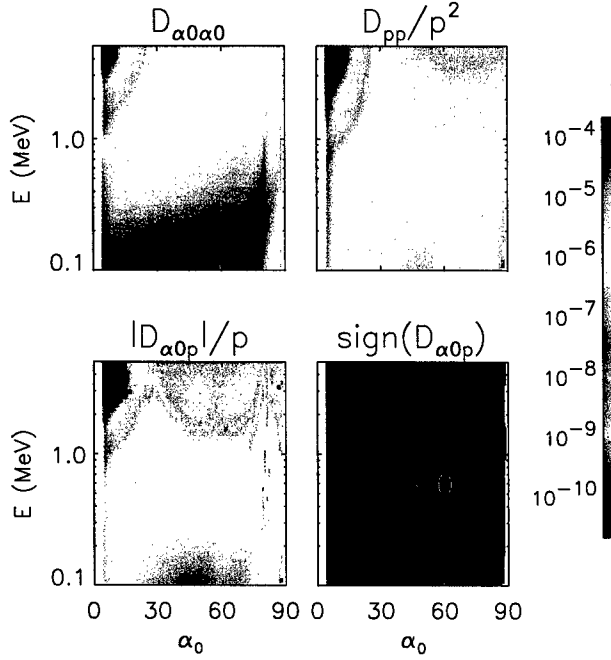


Figure 1. First three panels: inverse timescales from quasi-linear diffusion coefficients, in units of s^{-1} , for electrons at $L = 4.5$ based on a global model of storm time whistler-mode chorus waves. The last panel shows where the cross diffusion coefficient is positive (red) or negative (blue).

truly two dimensional, so that $\det(D_{JJ}) > 0$. This is strictly true for the bounce-averaged quasi-linear diffusion coefficients considered here, though barely so if the resonances are highly localized in both wave normal angle and latitude (as can occur for particles with $n = 0$ resonances only) [Albert, 2004].

[5] The slopes, dJ_2/dJ_1 , of curves on which Q_1 and Q_2 are constant are given, respectively, by

$$S_1 = -\frac{\partial Q_1/\partial J_1}{\partial Q_1/\partial J_2}, \quad S_2 = -\frac{\partial Q_2/\partial J_1}{\partial Q_2/\partial J_2}. \quad (4)$$

From equation (3), the requirement $\tilde{D}_{12} = 0$ can be expressed in terms of the slopes as

$$S_1 S_2 D_{11} - (S_1 + S_2) D_{12} + D_{22} = 0, \quad (5)$$

and the nonvanishing diffusion coefficients are

$$D_1 = (\partial Q_1/\partial J_2)^2 (S_1^2 D_{11} - 2S_1 D_{12} + D_{22}), \quad (6)$$

$$D_2 = (\partial Q_2/\partial J_2)^2 (S_2^2 D_{11} - 2S_2 D_{12} + D_{22}),$$

which are both positive since $D_{11} D_{22} > D_{12}^2$ [Albert, 2004]. In the new variables, the (J_1, J_2) part of the diffusion equation simplifies to

$$\frac{\partial f}{\partial t} = \frac{1}{G} \left(\frac{\partial}{\partial Q_1} G D_1 \frac{\partial f}{\partial Q_1} + \frac{\partial}{\partial Q_2} G D_2 \frac{\partial f}{\partial Q_2} \right), \quad (7)$$

where $G = \det[\partial(J_1, J_2)/\partial(Q_1, Q_2)]$ and there is no cross diffusion term. The radial diffusion term, if included, is unchanged.

[6] A second condition is required in order to determine the two slopes. One alternative is to impose $S_1 S_2 = -1$, so that the families of constant- Q_1 and constant- Q_2 curves are orthogonal in the (J_1, J_2) plane; this is equivalent to taking the constant- Q curves parallel to the eigenvectors of the D_{JJ} matrix. A simpler approach, adopted here, is to specify Q_1 as a convenient function of J_1 and J_2 , which determines S_1 and thus S_2 from equation (5).

[7] As a simple illustration, take the elements of D_{JJ} to be constant. The Green function solution of the (J_1, J_2) part of equation (1), for a distribution initially localized at (J_{10}, J_{20}) far from the boundaries, is

$$\exp[-(D_{22}\delta J_1^2 - 2D_{12}\delta J_1\delta J_2 + D_{11}\delta J_2^2)/4t\Lambda]/4\pi t\sqrt{\Lambda},$$

where $\delta J_1 = J_1 - J_{10}$, $\delta J_2 = J_2 - J_{20}$, and $\Lambda = D_{11}D_{22} - D_{12}^2$. Choosing $Q_1 \equiv J_2$ leads to $Q_2 = J_1 - (D_{12}/D_{22})J_2$ and $G = -1$, as well as $D_1 = D_{22}$ and $D_2 = D_{11} - D_{12}^2/D_{22}$. Then it may be checked that the Green function solution of equation (7), namely $\exp(-\delta Q_1^2/4D_1t - \delta Q_2^2/4D_2t)/4\pi t\sqrt{D_1 D_2}$, is the same as the expression above.

[8] Choosing Q_1 to be the equatorial pitch angle α_0 , the equation for constant- Q_2 curves can be written as

$$\frac{dE}{d\alpha_0} = \frac{D_{\alpha_0 E}}{D_{\alpha_0 \alpha_0}}, \quad (8)$$

and the transformed diffusion coefficients simplify to

$$D_1 = D_{\alpha_0 \alpha_0}, \quad D_2 = \left(\frac{\partial Q_2}{\partial E} \right)^2 \left(D_{EE} - \frac{D_{\alpha_0 E}^2}{D_{\alpha_0 \alpha_0}} \right). \quad (9)$$

The (α_0, E) diffusion coefficients and (D_{11}, D_{12}, D_{22}) are related by equation 3 so that, e.g., $D_{\alpha_0 \alpha_0} \sim (\Delta\alpha_0)^2/\Delta t$ rather than $(p\Delta\alpha_0)^2/\Delta t$ as by Lyons [1974a, 1974b]. As above, the expressions for the quasi-linear diffusion coefficients guarantee that D_2 is positive.

3. Performing the Transformation

[9] The diffusion coefficients ($D_{\alpha_0 \alpha_0}$, $D_{\alpha_0 p}/p$, D_{pp}/p^2 , all with units of s^{-1}) due to storm-time chorus were calculated at $L = 4.5$, using the wave models of Horne *et al.* [2005] and the computational methods of Albert [2005]. In these wave

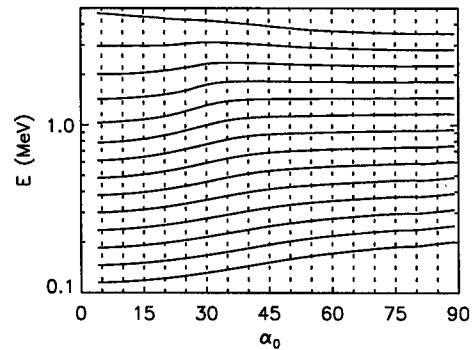


Figure 2. Curves of constant $Q_1 \equiv \alpha_0$ (dotted) and Q_2 (solid) corresponding to the diffusion coefficients of Figure 1, constructed so that the cross diffusion coefficient vanishes.

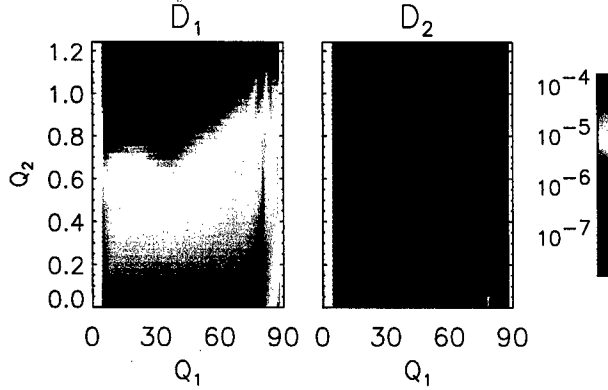


Figure 3. Diffusion coefficients D_1 and D_2 , in s^{-1} , for the variables Q_1 and Q_2 determined by the curves of Figure 2. Q_1 is chosen to be α_0 , and the constant- Q_2 curves are labeled by the value of $\log_{10}(E/0.2 \text{ MeV})$ at $\alpha_0 = 90^\circ$.

models, the wave power spectral density was taken to be the product of truncated Gaussians in frequency and wave normal angle ($\tan \theta$) in each of three different local time sectors. In the nighttime sector model (2300–0600 MLT), the waves lie between 0 and 15° latitude; the corresponding range of $f_{pe} f_{ce}$ is about 3.4 to 2.5. The waves are only present between 15° and 35° latitude in the prenoon sector model (0600–1200 MLT) and 10° to 35° latitude in the afternoon sector (1200–1500 MLT); the corresponding ranges of $f_{pe} f_{ce}$ are ~ 3.0 to 0.9 and 5.9 to 1.4 , respectively. The $\tan \theta$ distribution was further restricted to less than 0.9 times the resonance cone value, and resonance harmonics up to $n = \pm 5$ were included. The appropriately weighted, combined diffusion coefficients are shown in Figure 1,

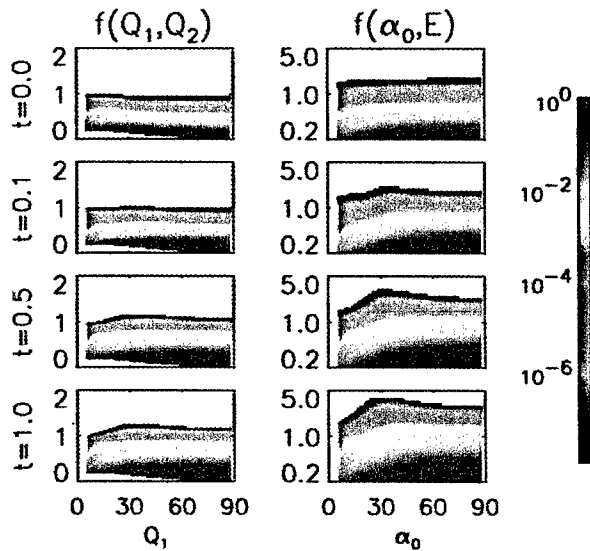


Figure 4. Phase space density (left) $f(Q_1, Q_2)$ and (right) $f(\alpha_0, E)$ at $t = 0, 0.1, 0.5$, and 1 day, from a numerical solution to the (Q_1, Q_2) diffusion equation. The lower boundaries in both columns correspond to $E = 0.2 \text{ MeV}$. The initial and boundary conditions are discussed in the text.

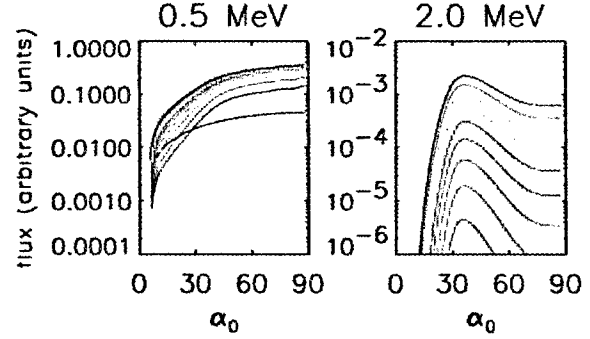


Figure 5. Electron flux as a function of α_0 at $E = 0.5 \text{ MeV}$ (left) and $E = 2 \text{ MeV}$ (right), at $t = 0$ (blue) to $t = 1$ day (red), at intervals of 0.1 day, corresponding to Figure 4.

and reflect the artificially sharp cutoffs used to model the frequency, wave normal angle, latitude, and local time distributions of wave power. Note that the cross diffusion coefficient, $D_{\alpha_0 p} \sim \langle \Delta \alpha_0 \Delta p / \Delta t \rangle$, can have either sign.

[10] The diffusion coefficients were used to numerically trace curves of constant Q_2 in the (α_0, E) plane, shown in Figure 2. (Since computation of the diffusion coefficients is quite demanding, equation (8) was integrated by interpolating a table of precomputed values; this also tended to smooth the discontinuities.) Diffusion in $Q_1 (= \alpha_0)$ proceeds along curves of constant Q_2 , which are roughly curves of constant energy (corresponding to the approximation of pure pitch angle diffusion). Diffusion in Q_2 proceeds along curves of constant Q_1 , which reach large values of energy.

[11] So far, the actual values of Q_2 have not been specified; they may be assigned in any smooth way that labels the curves on which they are constant. From Figure 2, it is natural to associate with each constant- Q_2 curve the value of E (more precisely, $\log_{10}[E/0.2 \text{ MeV}]$) near $\alpha_0 = 90^\circ$. Intersections of the constant- Q_1 curves and constant- Q_2 curves establish the correspondence between coordinates (Q_1, Q_2) and (α_0, E) . To evaluate D_1 and D_2 it is also necessary to evaluate the partial derivatives relating the old and new variables. A general procedure is to trace curves with constant but slightly differing values of Q_2 , which

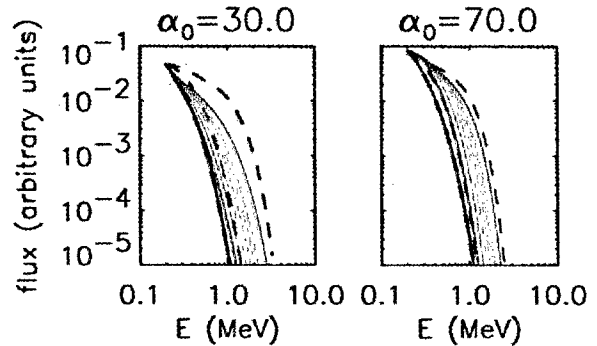


Figure 6. Electron flux $j = p^2 f$ as a function of E at (left) $\alpha_0 = 30^\circ$ and (right) $\alpha_0 = 70^\circ$ at $t = 0$ (blue) to $t = 1$ day (red), at intervals of 0.1 day, corresponding to Figure 4 (solid curves). Also shown are results at $t = 0, 0.1$, and 1 day obtained by neglecting cross diffusion $D_{\alpha_0 p}$ (dashed curves).

intersect a constant- Q_1 curve at slightly different values of (J_1, J_2) . This yields finite difference approximations for $\partial J_1/\partial Q_2$ and $\partial J_2/\partial Q_2$; and similarly for derivatives with respect to Q_1 . These determine the value of the Jacobian G along with $\partial Q_1/\partial J_1 = (\partial J_2/\partial Q_2)/G$, etc. However, with $Q_1 \equiv \alpha_0$, it is only necessary to numerically evaluate $\partial E/\partial Q_2$. The resulting values of D_1 and D_2 are shown in Figure 3.

4. Results and Discussion

[12] Using the values of D_1 and D_2 , equation (7) was solved numerically in the (Q_1, Q_2) plane, using fully explicit finite differencing for simplicity. To qualitatively model the depleted energetic electron levels that typically follow the main phase of a magnetic storm, the initial value of differential electron flux was taken to be $j = \exp [-(E - 0.2)/0.1] \sin \alpha_0$, with E in MeV, and $f = j/p^2$. These values were held fixed at the lower boundary, namely the curve where $E(Q_1, Q_2) = 0.2$ MeV, while $f = 0$ was imposed at the upper boundary. Zero flux was also enforced at the loss cone value of $Q_1 \equiv \alpha_0$, and $\partial f/\partial Q_1 = 0$ was taken as the boundary condition at $Q_1 = 90^\circ$.

[13] Results after 1 day are shown in Figure 4. Diffusion tends to transport phase space density to higher energy, but the outcome of the competition between this and diffusion into the loss cone could not be anticipated in detail. Pitch angle profiles, at constant energy, are shown in Figure 5. At 0.5 MeV the curves evolve roughly as expected on the basis of pitch angle diffusion alone, becoming peaked at $\alpha_0 = 90^\circ$ and falling rapidly near the edge of the loss cone. However, at 2 MeV the profiles develop a persistent peak around 35° , for which the energy diffusion rate is largest. Figure 6 shows one-dimensional flux vs. energy profiles developing in time. Significant levels of flux develop at 1 MeV in less than half a day. Energization is faster at $\alpha_0 = 30^\circ$ than at 70° , as expected from Figure 1, and reaches ~ 2 MeV during the 1 day of the simulation.

[14] To gauge the effect of cross diffusion, the entire procedure was repeated with $D_{\alpha_0 p}$ set to 0. The results are shown in Figure 6 as dashed curves at $t = 0, 0.1$, and 1 day. The qualitative behavior is similar, but for small α_0 neglecting cross diffusion evidently leads to an overestimate of energy diffusion (contrary to a conjecture by Horne *et al.* [2005]). It seems likely that $D_{\alpha_0 p}^2/(D_{\alpha_0 \alpha_0} D_{pp})$ is a controlling parameter, based on the Green function expressions.

[15] Several additional physical processes must still be included. Scattering by electromagnetic ion cyclotron waves is likely a strong loss mechanism for electrons above 1 MeV, especially combined with whistler mode hiss [Summers *et al.*, 1998; Summers and Thorne, 2003; Albert, 2003]. Radial diffusion, particularly when enhanced during storm times, is a potentially crucial source of energization [O'Brien *et al.*, 2003] and also serves to redistribute locally accelerated particles [Green and Kivelson, 2004]. The framework presented here should help make it possible to realistically combine and evaluate the effects of diffusion in all three adiabatic invariants.

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